Draft version March 1, 2019
Typeset using LaTeX **twocolumn** style in AASTeX62

THE HALO DRIVE: FUEL-FREE RELATIVISTIC PROPULSION OF LARGE MASSES VIA RECYCLED BOOMERANG PHOTONS

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ABSTRACT

Gravitational slingshots around a neutron star in a compact binary have been proposed as a means of accelerating large masses to potentially relativistic speeds. Such a slingshot is attractive since fuel is not expended for the acceleration, however it does entail a spacecraft diving into close proximity of the binary, which could be hazardous. It is proposed here that such a slingshot can be performed remotely using a beam of light which follows a boomerang null geodesic. Using a moving black hole as a gravitational mirror, kinetic energy from the black hole is transferred to the beam of light as a blueshift and upon return the recycled photons not only accelerate, but also add energy to, the spacecraft. It is shown here that this gained energy can be later expended to reach a terminal velocity of approximately 133% the velocity of the black hole. A civilization could exploit black holes as galactic way points but would be difficult to detect remotely, except for an elevated binary merger rate and excess binary eccentricity.

Keywords: relativistic processes — space vehicles — black holes

1. INTRODUCTION

In recent months, there has been increased interest in light sailing propulsion systems, including using direct energy, thanks (in part) to the *Breakthrough Starshot* project announced in 2016. Since the early 20$^{\text{th}}$ century, it has been recognized that the momentum carried by light could be used to accelerate spacecraft (Zander 1925). Although the momentum exchanges are tiny, what makes radiation pressure attractive as a propulsion system is the fact that fuel need not be carried by the spacecraft itself. Either through Solar radiation (Garwin 1958) or directed energy (Marx 1966; Redding 1967; Forward 1984), such systems could be used to overcome the limitations imposed by the Tsiolkovsky rocket equation affecting conventional reaction drives.

Achieving relativistic speeds through such a system is theoretically achievable by directing high powered lasers at spacecraft (see Bible et al. 2013; Benford 2013). For non-relativistic speeds, the energy required to accelerate a spacecraft of mass m to velocity βc equals $\frac{\beta}{2}mc^2$ (via a first-order expansion in β of Equation (6) of Kulkarni et al. 2016). This highlights that accelerating massive objects to relativistic speeds is certainly challenging since one needs to first store energy comparable to the rest mass. Accelerating a low-mass ($\sim$ gram) spacecraft may be feasible albeit at considerable energetic cost ($\sim 10\,$TJ), but larger masses pose severe challenges.

An idealized propulsion system would be able to accelerate arbitrarily large masses to relativistic speeds at little to no energy cost. At first, this statement may seem fanciful yet essentially free speed-boosts have been exploited for decades in the Solar System via gravitational assists, although not to the speeds associated with relativistic flight. Perhaps the ultimate incarnation of the gravity assist was proposed by Dyson (1963), who argued that a compact binary of white dwarfs or neutron stars could be exploited to accelerate arbitrarily large masses up to relativistic speeds (assuming the binary is sufficiently compact). This “Dyson slingshot” maneuver is theoretically attractive but swinging round a neutron star in close proximity is potentially hazardous due to extreme tidal forces and the circumbinary radiation environment.

In this work, it is shown that the Dyson slingshot can be performed remotely using the ideas from directed energy light sailing and gravitational mirrors. Gravitational mirrors were first described in Stuckey (1993) who

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showed that null geodesics exists around Schwarzschild black holes enabling one to see one's own reflection. Photons just skimming the photon sphere perform a full revolution and can make their way back to the source, dubbed as "boomerang photons" by the author. Using this effect, it is argued here that a moving black hole can be used like a moving mirror, causing light to not only return to the source but also receive a blue shift due to the black hole's relative motion. Photons are recycled by the spacecraft and repeatedly emitted and re-absorbed from the gravitational mirror, accelerating the spacecraft up to speeds ultimately exceeding that of the black hole itself.

For convenience, this setup is referred as a "halo drive" in what follows, as a result of the ring of light which wraps around the black hole and the propulsive nature of the final outcome. Such a system is capable of achieving the Dyson slingshot but without requiring a spacecraft to become in close proximity of the binary itself. Whilst slingshots could be performed around an isolated moving black hole, binaries are focused on in this work due to their potential for compact configurations where relativistic speeds could be achieved (although the expressions derived throughout are equally applicable to isolated black holes too). With $\mathcal{O}[10^8]$ black holes estimated to reside within the Milky Way (Elbert et al. 2017), a large network of way-points potentially exist to permit intra-galactic travel. This work describes some of the basic mathematics behind the halo drive concept and the consequences for both the spacecraft and the binary itself.

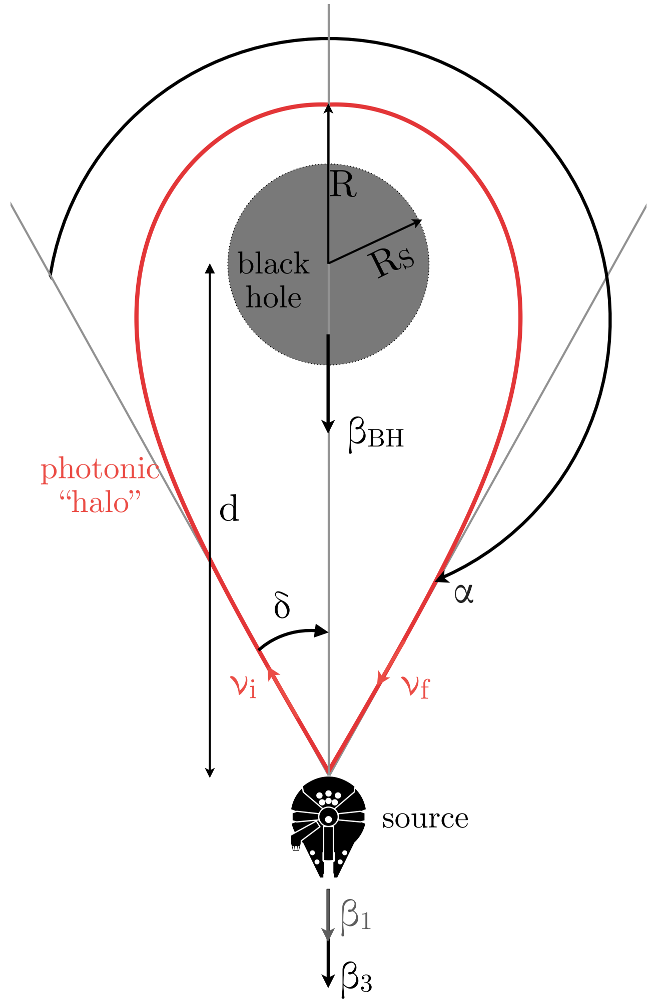

Figure 1. Outline of the halo drive. A spaceship traveling at a velocity β_i emits a photon of frequency ν_i at a specific angle δ such that the photon completes a halo around the black hole, returning shifted to ν_f due to the forward motion of the black hole, β_{BH}.

2. DEFLECTION OFF A MOVING BLACK HOLE

2.1. *A halo from boomerang photons*

The majority of this paper will concern itself will computing the velocities which can be achieved by a spacecraft using the halo drive described in Section 1. However, it is worth first establishing that boomerang photon geodesics exist and considering the shape of such paths.

Boomerang null geodesics were first introduced by Stuckey (1993), who considered the Schwarzschild metric and demonstrated that such geodesics exist and effectively turn black holes in gravitational mirrors. Indeed, theoretically an infinite number of distinct boomerang geodesics exist, corresponding to how many loops around the black hole are conducted. If one writes the critical impact parameter for photon capture as b_c, then the number of loops of a scattered photon equals $N \sim -\log(-1 + b/b_c)/2\pi$ (Zeldovich & Novikov 1971). However, for the sake of this work, the scope is limited to that of the first-order geodesic which does not perform multiple revolutions.

Light is emitted from the source at an angle δ relative to the radial direction and experiences strong deflection as it approaches the event horizon. For a Schwarzschild boomerang geodesic, there is rotational symmetry about the radial direction meaning that the angle of emission equals the angle of reception (Stuckey 1993). The basic setup is depicted in Figure 1.

In order for the deflection to be strong enough to constitute a boomerang, this requires the light's closest approach to the black hole to be within a couple of Schwarzschild radii, $R_S \equiv 2GM/c^2$. Light which makes a closest approach smaller than $3GM/c^2$ becomes trapped in orbit, known as the photon sphere, and thus typical boomerang geodesics skim just above this critical distance.

Stuckey (1993) showed that boomerang null geodesics could be computed by numerically integrating the rate of change of the radial coordinate with respect to the azimuthal coordinate, $\mathrm{d}r/\mathrm{d}\varphi$ (a simple algorithm is described in the Appendix of that work). To illustrate this, numerical integrations of the geodesic were performed with 10^6 steps for a series of different initial standoff distances, d. As shown in Figure 2, the critical deflection necessary to perform a boomerang appears to be proportional to $1/d$ to a good approximation (particularly when $d \gg GM/c^2$), with a constant of proportionality given by $\delta_0 = 286.5°$.

More importantly for this work, the experiment described above demonstrates that $\delta \to 0$ as d becomes large and thus the angle α depicted in Figure 1 approaches π radians (since $\alpha = \pi - 2\delta$). This simplification will be exploited this later in Section 3.

Even in the idealized Schwarzschild case, the results shown above are not generally applicable to a practical halo drive. This is because the photon should not return to the precise same location but rather a greater radial distance, since the spacecraft will experience a back-reaction after emission (or exhaust) of the photon. Thus, the angle should be chosen according to the rate of acceleration desired.

For the sake of demonstrating the principle of the halo drive, this paper does not concern itself with the precise angular correction needed to accomplish this. It is worth highlighting that boomerang geodesics can be constructed in the more general case of a Kerr metric, as discussed in Cramer (1997). Since the halo drive exploits a compact binary, both components should be included in a more precise calculation. For the sake of this work, it is sufficient to note that such a) geodesics exist and are computable b) the angle $\alpha \simeq \pi$ when $d \gg GM/c^2$, simplifying subsequent calculations. Note that point a) could be calculated onboard the spacecraft either using a metric known to be completely correct, or using a pilot low-power laser to fine-tune the correct angle.

2.2. *Deflections in the black hole's rest frame*

Let us now turn to calculating the movement of a spacecraft in response to emitting a boomerang photon (or halo) around a moving black hole. Before considering the effect on the spacecraft, one needs to first derive the changes imparted onto a photon which conducts such a loop.

Let us work in the rest frame of the black hole and assuming that an incident photon passes by with an impact parameter exceeding $3\sqrt{3}GM/c^2$, such that the closest approach exceeds $3GM/c^2$. In such a case, it is expected that the photon to be deflected by some arbitrary angle (see Darwin 1959) which is labelled as α', as depicted in Figure 3.

As described in Section 2.1, the angle α' is set by the mass of the BH and the impact parameter of the encounter. To start, let us consider what the frequency of the deflected photon, ν'_f, will be. This can be computed by conserving relativistic four-momentum before and after the encounter. The initial four-momentum of the photon, working in units of c, is given by

$$\mathbf{P}_{\mathrm{EM,pre}} = \begin{pmatrix} h\nu'_i \\ 0 \\ h\nu'_i \\ 0 \end{pmatrix}, \tag{1}$$

and that of the black hole of

$$\mathbf{P}_{\mathrm{BH,pre}} = \begin{pmatrix} M \\ 0 \\ 0 \\ 0 \end{pmatrix}. \tag{2}$$

Let us write the final four-momentum vectors of these components as

$$\mathbf{P}_{\mathrm{EM,post}} = \begin{pmatrix} h\nu'_f \\ h\nu'_f \sin\alpha' \\ h\nu'_f \cos\alpha' \\ 0 \end{pmatrix}, \tag{3}$$

and

$$\mathbf{P}_{\mathrm{BH,post}} = \begin{pmatrix} E \\ p\sin\theta \\ p\cos\theta \\ 0 \end{pmatrix}. \tag{4}$$

Conserving each component of the total four-momentum, one finds

$$h\nu'_i + M = h\nu'_f + E, \tag{5}$$
$$0 = h\nu'_f \sin\alpha' + p\sin\theta, \tag{6}$$
$$h\nu'_i = h\nu'_f \cos\alpha' + p\cos\theta. \tag{7}$$

Taking the last two lines, then squaring and summing, one may write

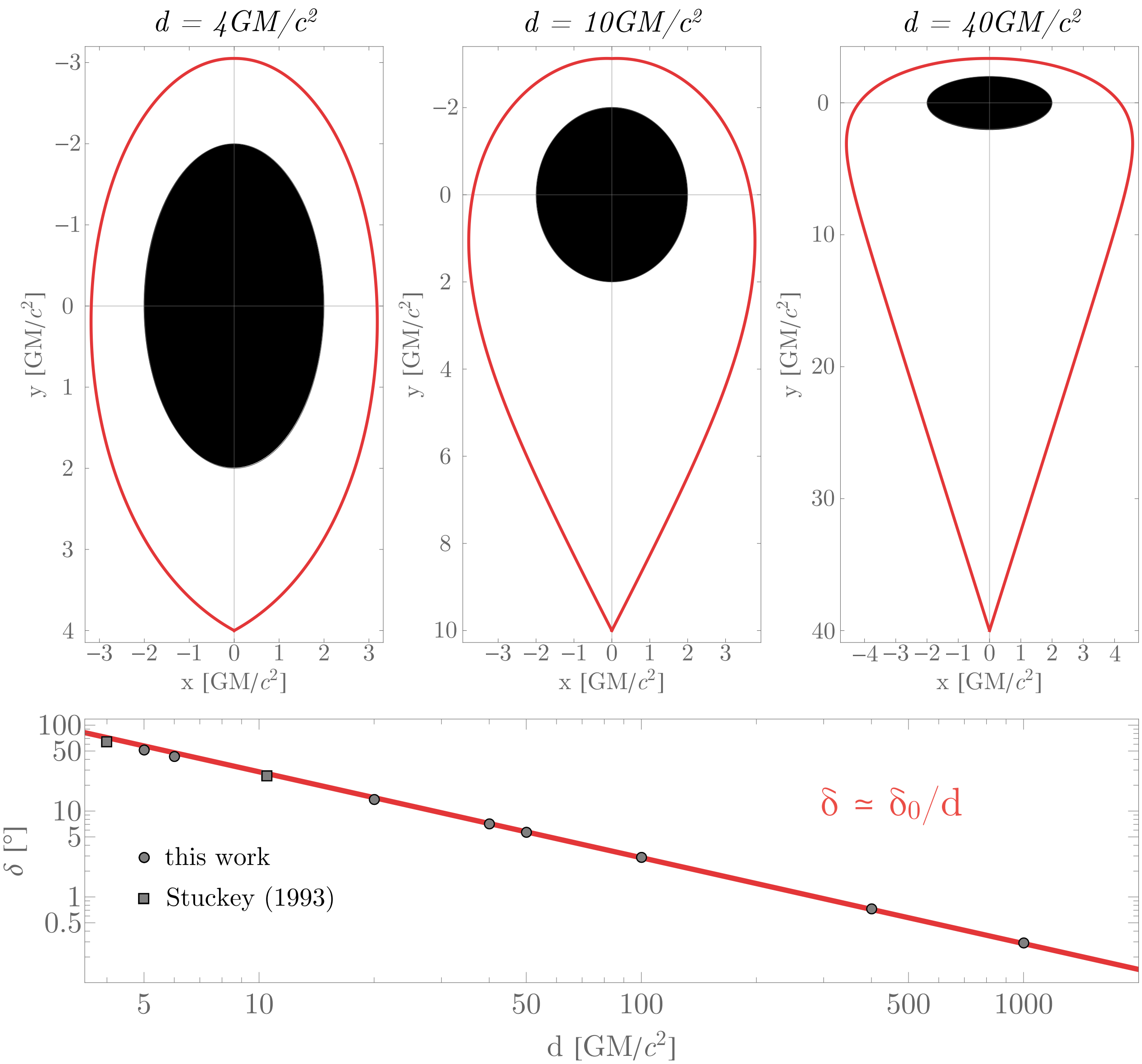

Figure 2. Top panel shows three numerically integrated boomerang null geodesics where the initial standoff distance, d, is varied. Solving for the boomerang deflection angle for a series of different d values, it may be seen that the angle drops off as $\propto 1/d$ to a good approximation, especially when $d \gg GM/c^2$.

$$p^2 = (h\nu'_f \sin\alpha')^2 + (h\nu'_i - h\nu'_f \cos\alpha')^2. \tag{8}$$

One may now use the relation $E^2 = p^2 + M^2$ and our earlier energy expression to write that

$$(h\nu'_i + M - h\nu'_f)^2 = (h\nu'_f \sin\alpha')^2 + (h\nu'_i - h\nu'_f \cos\alpha')^2 + M^2. \tag{9}$$

Solving the above for ν'_f yields the familiar Compton scattering equation

$$\nu'_f = \frac{\nu'_i}{1 + \frac{h\nu'_i}{Mc^2}(1 - \cos\alpha')}, \tag{10}$$

where the c units have been re-added.

2.3. *Deflections around a moving black hole*

The general principle of the halo drive is to siphon kinetic energy from the BH and thus one ultimately requires computing deflections around a moving BH. Armed with the result from the previous subsection, this can be easily accomplished using Lorentz transforms.

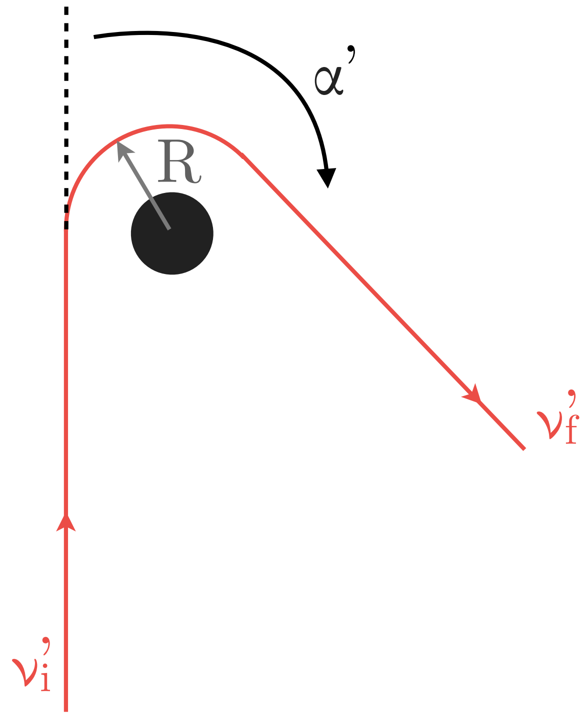

Figure 3. An incident photon of frequency ν_i' is deflected around a Schwarzschild black hole by an angle α', depicted here in the rest frame of the BH.

Let us assume that the black hole is moving along the $-\hat{y}$ direction such that its velocity vector is given by $\mathbf{v} = \{0, -\beta_{\mathrm{BH}}c, 0\}^T$. In such a case, the incident photon's energy in the observer's frame, ν_i, may be related to that in the black's hole (initial) rest frame, ν_i', using

$$\nu_i' = \nu_i \sqrt{\frac{1+\beta_{\mathrm{BH}}}{1-\beta_{\mathrm{BH}}}}. \tag{11}$$

The deflected photon returns at an arbitrary angle and so using the more general boosting expression yields

$$\nu_f = \nu_f' \frac{\sqrt{1-\beta_{\mathrm{BH}}^2}}{1+\beta_{\mathrm{BH}}\cos\alpha}. \tag{12}$$

One may now account for the photon's deflection by substituting ν_f' using Equation (10) to obtain

$$\nu_f = \nu_i \frac{\sqrt{\frac{1+\beta_{\mathrm{BH}}}{1-\beta_{\mathrm{BH}}}}}{1+\sqrt{\frac{1+\beta_{\mathrm{BH}}}{1-\beta_{\mathrm{BH}}}}\frac{h\nu_i}{Mc^2}(1-\cos\alpha')} \frac{\sqrt{1-\beta_{\mathrm{BH}}^2}}{1+\beta_{\mathrm{BH}}\cos\alpha}. \tag{13}$$

It is worth highlighting that in the limit of $M \to \infty$ (an infinite mass mirror) and $\alpha \to \pi$ (a normal reflection), Equation (13) reproduces same familiar result as that of (Einstein 1905), i.e.

$$\lim_{M\to\infty} \alpha \to \pi \nu_f = \nu_i \Big(\frac{1+\beta_{\mathrm{BH}}}{1-\beta_{\mathrm{BH}}}\Big). \tag{14}$$

Finally, one needs to relate α', the angle of deflection in the black hole's initial rest frame, to α, the angle of deflection in the observer's frame. Consider the setup as depicted in Figure 3 where α' is obtuse. If one defines an angle $\theta' = \pi - \alpha$ as the acute and opposite angle, this angle along the direction of motion should be expected to be squeezed in the moving frame as a result of relativistic aberation. Accordingly, one would expect the observer's frame to have $\theta' > \theta$, or more explicitly using the aberation formula

$$\cos\theta = \frac{\cos\theta' - \beta_{\mathrm{BH}}}{1-\beta_{\mathrm{BH}}\cos\theta'}. \tag{15}$$

Re-writing in terms of α angles and re-arranging to make α' the subject, one finds

$$\cos\alpha' = \frac{\beta_{\mathrm{BH}} - \cos\alpha}{\beta_{\mathrm{BH}}\cos\alpha - 1}. \tag{16}$$

Plugging this into our earlier result for the frequency shift given by Equation (13), one may write

$$\nu_f = \nu_i \Bigg(\frac{1+\beta_{\mathrm{BH}}}{1+\beta_{\mathrm{BH}}\cos\alpha}\Bigg)\Bigg(\frac{1}{1+K_i\sqrt{\frac{1+\beta_{\mathrm{BH}}}{1-\beta_{\mathrm{BH}}}}(1-\frac{\beta_{\mathrm{BH}}-\cos\alpha}{\beta_{\mathrm{BH}}\cos\alpha-1})}\Bigg), \tag{17}$$

where

$$K_i \equiv \frac{h\nu_i}{Mc^2}. \tag{18}$$

In the limit where $h\nu_i \ll Mc^2$ (the infinite mass limit), then the photon's frequency upon return is well-approximated by

$$\lim_{K_i\to 0} \nu_f = \nu_i \Big(\frac{1+\beta_{\mathrm{BH}}}{1+\beta_{\mathrm{BH}}\cos\alpha}\Big). \tag{19}$$

2.4. *Gravitational redshifting during the deflection*

One effect that has been ignored thus far is gravitational blue/red shift. If the photon is assumed to return to the same location it originated from, then the net change in gravitational potential energy from emission to reflection is zero. However, during the approach of the photon, it will become increasingly blue, potentially affecting our expressions. It is argued here that this effect is extremely small and can be safely ignored for the purposes of this paper, although could be accounted for using numerical integrations.

Let us take the quite reasonable assumption that $Mc^2 \gg h\nu_i$, such that Equation (17) can be approximated to Equation (19). Let us denote the radial separation of the photon from the black hole as $d[t]$ i.e. as a function of time. Therefore, during the approach, one expects the photon's frequency to be blue shifted as

$$\nu_i[t] = \nu_i \sqrt{\frac{d[t](d_0 - R_S)}{d_0(d[t] - R_S)}}. \tag{20}$$

In other words, it is simply a multiplicative factor of the original frequency. The photon is then shifted by the deflection encounter according to Equation (19), which is again simply a multiplicative factor of frequency. Finally, upon return the equal and opposite gravitational redshift occurs (since the photon returns to the same location), cancelling out the previous blue shift.

Accordingly, in the limit of $Mc^2 \gg h\nu_i$ and the photon returning to the same location, gravitational blue/red shift has zero net effect. This symmetry is broken when one includes the K_i term, and again this could be correctly accounted for using numerical integrations, however it is a fairly extreme case that is technically forbidden as long as the spaceship has a low mass compared to the BH i.e. $m \ll M$ (see Section 3.5 for justification).

If the photon does not return to the same location but at a greater radial distance (as expected since the spacecraft will be in motion), then there will be a net effect even in the limit of $Mc^2 \gg h\nu_i$. This is discussed later in Section 3.5.

3. FORMALISM FOR THE HALO DRIVE

3.1. *Response of a spacecraft*

Consider an initial setup where a spacecraft of mass m_1 resides in a wide orbit around a binary BH. At one of the quadrature point in the binary orbit, the BH will be approaching the spacecraft at a relative velocity of β_{BH}. More generally, the spacecraft may have already begun to accelerate away from the black hole and thus have a velocity of β_1 in the same direction.

The source (or spacecraft) emits a photon of energy ν_i and this will lead to a slight back impulse on the source. The source must also slightly decrease in mass as a result of the emission, reducing from m_1 to m_2, culminating in the source increasing in speed from β_1 to β_2. Conserving relativistic energy and momentum, one may write that

$$\begin{aligned} \frac{m_1 c^2}{\sqrt{1-\beta_1^2}} &= h\nu_i + \frac{m_2 c^2}{\sqrt{1-\beta_2^2}}, \\ \frac{m_1 c \beta_1}{\sqrt{1-\beta_1^2}} &= -\frac{h\nu_i}{c} + \frac{m_2 c \beta_2}{\sqrt{1-\beta_2^2}}, \end{aligned} \tag{21}$$

where m is the mass of the source. Solving the above and simplifying, one finds a speed of

$$\beta_2 = \frac{\beta_1 + \kappa_{i1}\sqrt{1-\beta_1^2}}{1 - r_{i1}\sqrt{1-\beta_{i1}^2}}, \tag{22}$$

and a mass of

$$\frac{m_2}{m_1} = \sqrt{1 - 2\kappa_{i1}\sqrt{\frac{1+\beta_1}{1-\beta_1}}}, \tag{23}$$

where in both expressions

$$\kappa_{i1} \equiv \frac{h\nu_i}{m_1 c^2}. \tag{24}$$

The spacecraft has finite mass and so cannot emit photons of arbitrary energy. Taking the resulting equation for m_2/m_1, one may solve that the limiting case is when the mass approaches zero, the maximum allowed photon emission corresponds to

$$\kappa_{i1,\mathrm{max}} = \frac{1}{2}\sqrt{\frac{1-\beta_1}{1+\beta_1}}. \tag{25}$$

In this where the mass is totally converted into energy, the final speed of the now massless spacecraft can be shown to equal c, since it is essentially just the returning photon.

The results can be combined with the change once the photon returns with a modified frequency ν_f. Consider the simplified case where the final and initial position of the source are both sufficiently out of the gravitational well that the effects of gravitational redshift can be ignored. Further, the relativistic Doppler effect that occurs between the returning photon and the now-moving spacecraft is ignored, such that ν_f is given by Equation (19). The Doppler effect will be accounted for later in Section 3.4. Under these assumptions, one can construct another set of equations for the absorption given by

$$\frac{m_2 c^2}{\sqrt{1-\beta_2^2}} + h\nu_f = \frac{m_3 c^2}{\sqrt{1-\beta_3^2}},$$
$$\frac{m_2 c\beta_2}{\sqrt{1-\beta_2^2}} + \frac{h\nu_f}{c} = \frac{m_3 c\beta_3}{\sqrt{1-\beta_3^2}}, \tag{26}$$

giving

$$\beta_3 = \frac{\beta_2 + \kappa_{f2}\sqrt{1-\beta_2^2}}{1+\kappa_{f2}\sqrt{1-\beta_2^2}}, \tag{27}$$

and

$$\frac{m_3}{m_2} = \sqrt{1 + 2\kappa_{f2}\sqrt{\frac{1-\beta_2}{1+\beta_2}}}, \tag{28}$$

where in both expressions

$$\kappa_{f2} \equiv \frac{h\nu_f}{m_2 c^2}. \tag{29}$$

Note that the latter of the new equations reveals that $\lim_{m_2\to 0} m_3 = 0$, which happens when $\kappa_{i1} \to \kappa_{i1,\mathrm{max}}$. In other words, if the spacecraft converts all of its mass into energy and returns as a pure photon, there is no mechanism here for the photon to somehow return back to massive spacecraft. Substituting in the earlier equations, and after much simplification, one finds that

$$\lim_{K_i\to 0}\lim_{\alpha\to\pi}\beta_3 = \frac{\beta_1(1-\beta_{\mathrm{BH}}) + 2\kappa_{i1}\sqrt{1-\beta_1^2}}{(1-\beta_{\mathrm{BH}}) + 2\kappa_{i1}\beta_{\mathrm{BH}}\sqrt{1-\beta_1^2}}. \tag{30}$$

In the limit of the photon's carrying no momentum ($\kappa_{i1} \to 0$), then the final velocity is unchanged from the initial velocity, as expected. In the limit of the intermediate mass, m_2, being zero implying a complete conversion into energy, the final speed is c as expected for a massless particle. It is worth highlighting that in the limit of $\kappa_{i1} \to 0$, which is to say the back-reaction effect described in Kipping (2017) is ignored, then $\beta_3 \to \beta_1$ and no acceleration is achieved, underlining the importance of the effect described in that paper.

Although the velocity change in Equation (30) is small for low choices of κ_{i1}, it is emphasized that any number of photons can be fired and at any frequency and these velocity differences accumulate. At each stage, not only is the source accelerated, but it is also gains mass (or energy). Specifically, the mass change is given by

$$\lim_{K_i\to 0}\lim_{\alpha\to\pi}\frac{m_3}{m_1} = \sqrt{1 - 2\kappa_{i1}\sqrt{\frac{1+\beta_1}{1-\beta_1}}}\sqrt{1 + 2\kappa_{i1}\sqrt{\frac{1-\beta_1}{1+\beta_1}}\Big(\frac{1+\beta_{\mathrm{BH}}}{1-\beta_{\mathrm{BH}}}\Big)}. \tag{31}$$

Note that m_3 equals m_1 if $\kappa_{i1} \to 0$, demonstrating again that if the Kipping (2017) back-reaction effect is ignored, the mass of the spacecraft would be unchanged not allowing for any energy gains. Further, it is noted that in the limit of $\beta_1 \to 0$ and $\beta_{\mathrm{BH}} \to 0$, no mass gain should be possible and indeed this is apparent since the solution becomes $m_3 = m_1\sqrt{1-4\kappa_{i1}^2}$ i.e. $m_3 < m_1$ for all $\kappa_{i1} > 0$.

3.2. *Equilibrium velocity*

Consider starting at rest, $\beta_1 = 0$, and emitting a photon which gives a final velocity such that the spaceship ends up with a maximally increased mass. This can be calculated by taking the limit of Equation (31) for $\beta_1 \to 0$ and then differentiating $\partial[\lim_{\beta_1\to 0} m_3]/\partial\kappa_{i1} = 0$ solving for κ_{i1}. This occurs when

$$\kappa_{i1} = \frac{1}{2}\Big(\frac{\beta_{\mathrm{BH}}}{1+\beta_{\mathrm{BH}}}\Big), \tag{32}$$

giving a final mass of

$$\frac{m_3}{m_1} = \gamma_{\mathrm{BH}}, \tag{33}$$

where $\gamma_{\mathrm{BH}} = (1-\beta_{\mathrm{BH}}^2)^{-1/2}$. Evaluating the corresponding velocity, which is labelled as the "equilibrium velocity" in what follows (β_{eq}):

$$\beta_{\mathrm{eq}} = \beta_{\mathrm{BH}}, \tag{34}$$

which has an intuitive interpretation since at parity speed $\nu_f = \nu_i$.

3.3. *Terminal velocity*

This gained mass can now be used to induce further acceleration. Whilst this could be achieved by simply exhausting photons, the most efficient means would be again to use the BH mirror and exploit the halo drive.

Let us take Equation (31), set $\beta_1 \to \beta_{\rm BH}$ since the starting speed is the equilibrium speed, and solve the expression to be equal to $1/\gamma_{\rm BH}$ with respect to κ_{i1}. The photon energy needed is easily found to be given by

$$\kappa_{i1} = \frac{\beta_{\rm BH}}{2}\sqrt{\frac{1-\beta_{\rm BH}}{1+\beta_{\rm BH}}}, \tag{35}$$

and plugging into our β_3 equation where β_1 is to again initiated from $\beta_{\rm BH}$, one obtains a "terminal velocity", $\beta_{\rm term}$, of

$$\beta_{\rm term} = \frac{2\beta_{\rm BH}}{1+\beta_{\rm BH}^2}, \tag{36}$$

which is bound to be $0 \leq \beta_{\rm term} < 1$ for all $0 \leq \beta_{\rm BH} < 1$, as expected. Expanding to third-order in $\beta_{\rm BH}$, $\beta_{\rm term}$ may be written as

$$\beta_{\rm term} = 2\beta_{\rm BH} - 2\beta_{\rm BH}^3 + \mathcal{O}[\beta_{\rm BH}^5]. \tag{37}$$

To first order then, the terminal velocity equals twice that of the black hole, consistent with the first-order result for a conventional gravitational slingshot. In essence then, one is conducting a remote slingshot using the halo rather than physically approaching the BH and risk tidal disruption (as well as an increased flight time and heavy time dilation by diving into the gravitational well).

From Kipping (2017), one expects the following two statement to be true, if the principle of ensemble equivalence holds. First, rather than accelerating from rest to equilibrium speed with one photon, and then equilibrium to terminal with a second photon, the same acceleration could be achieved for the same energy using a large number of smaller photon energies. This point is important because it is impractical to emit such a high energy photon in a single step. Second, if this principle holds, then the reverse should also be true and both steps should be achievable in a single photon i.e. one should be able to accelerate from rest to terminal with a single photon emission.

This latter statement may be verified by solving $\lim_{\beta_1 \to 0} m_3 = m_1$ with respect to κ_{i1} - the single high energy photon, which yields a quadratic solution of

$$\kappa_{i1} = \begin{cases} 0, \\ \frac{\beta_{\rm BH}}{1+\beta_{\rm BH}} \end{cases}. \tag{38}$$

The zero result clearly corresponds to no motion at all. Plugging the latter result into our β_3 equation in the limit where $\beta_1 \to 0$ yields the same terminal velocity as that stated in Equation (36), in accordance with the principle.

It's worth comparing this single photon emission to that of the maximum photon emission earlier, $\kappa_{i1,\rm max}$. One may easily show that $\lim_{\beta_1 \to 0} \kappa_{i1,\rm max}$ equals this single photon energy if, and only if, $\beta_{\rm BH} = 1$. This therefore re-enforces that this physical limit cannot be practically achieved.

3.4. *Accounting for relativistic Doppler shifts*

One important effect thus far ignored is the relativistic Doppler shift of the returning photon in the spacecraft's frame of motion. Even for a single photon emission, the emission causes a back-reaction which accelerates the spacecraft away from rest up to β_2. Accordingly, when the photon returns it is not reabsorbed as ν_f but as ν_f'', where the dashes indicate a Lorentz transform to the rest frame of the spacecraft.

Following the principle of photon equivalence, one can simplify the derivation by considering a single photon emission to accelerate up to terminal velocity - defined as the maximum speed for which $m_3 = m_1$. Starting from rest, β_2 and m_2 are the same as Equations (22) & (23) found earlier, except that $\beta_1 \to 0$, giving

$$\lim_{\beta_1 \to 0} \beta_2 = \frac{\kappa_{i1}}{1-\kappa_{i1}}, \tag{39}$$

$$\lim_{\beta_1 \to 0} \frac{m_2}{m_1} = \sqrt{1-2\kappa_{i1}}. \tag{40}$$

Before, it was assumed that the photon returned with a frequency given by Equation (19) in the limit of $\alpha \to \pi$. One may now modify this to

$$\nu_f = \nu_i \Big(\frac{1+\beta_{\rm BH}}{1-\beta_{\rm BH}}\Big)\sqrt{\frac{1-\beta_2}{1+\beta_2}}, \tag{41}$$

where the square root term accounts for the relativistic Doppler shift. When this photon returns, the final mass, m_3, can be calculated using Equation (28) except the photon energy is substituted using Equation (41), yielding

$$\frac{m_3}{m_1} = (1-2\kappa_{i1})\Bigg(1 + 2\kappa_{i1}\sqrt{1-2\kappa_{i1}}\Big(\frac{1+\beta_{\rm BH}}{1-\beta_{\rm BH}}\Big)\Bigg). \tag{42}$$

Solving $m_3 = m_1$ with respect to κ_{i1} yields

$$\kappa_{i1} = \frac{1}{2}\left(1 - \left(\frac{1-\beta_{\rm BH}}{1+\beta_{\rm BH}}\right)^{2/3}\right). \quad (43)$$

Plugging this result into Equation (27) yields a revised terminal velocity (after much simplification) of

$$\beta_{\rm term} = \frac{(1+\beta_{\rm BH})^{4/3} - (1-\beta_{\rm BH})^{4/3}}{(1+\beta_{\rm BH})^{4/3} + (1-\beta_{\rm BH})^{4/3}}, \quad (44)$$

which is again bound to be $0 \leq \beta_{\rm term} < 1$ for all $0 \leq \beta_{\rm BH} < 1$. Expanding to third-order for small $\beta_{\rm BH}$, $\beta_{\rm term}$ may be written as

$$\beta_{\rm term} = \frac{4}{3}\beta_{\rm BH} - \frac{28}{81}\beta_{\rm BH}^3 + \mathcal{O}[\beta_{\rm BH}^5]. \quad (45)$$

In the limit of large $\beta_{\rm BH}$, Equation (44) is well-approximated by $\gamma_{\rm term} \simeq 2^{1/3}\gamma_{\rm BH}^{4/3}$. These results show that the Doppler shifts decrease the amount of energy transferred to the spacecraft, but nevertheless speeds in excess of the black hole's velocity can be achieved.

3.5. *Accounting for gravitational red/blue-shifts*

As discussed earlier in Section 2.4, gravitational red/blue shifts can be shown to be an extremely small effect so long as $Mc^2 \gg h\nu_i$ and the photon returns to the same radial distance. According to Equation (25), $\kappa_{i1} < \frac{1}{2}$ and thus $mc^2 < h\nu_i/2$. Accordingly, the valid regime can also be stated as $M \gg m$. However, since the objective of the halo drive is to accelerate the spacecraft to relativistic velocities, then clearly the geodesic will be chosen such that the photon does not in fact return to the same location but rather a greater radial distance.

Consider starting from rest and attempting to accelerate to terminal velocity with a single photon of energy given by Equation (43). The intermediate velocity of the spacecraft is β_2, which here can be evaluated to be

$$\beta_2 = \frac{\kappa_{i1}}{1+\kappa_{i1}}, \\ = \frac{1 - \left(\frac{1-\beta_{\rm BH}}{1+\beta_{\rm BH}}\right)^{2/3}}{1 + \left(\frac{1-\beta_{\rm BH}}{1+\beta_{\rm BH}}\right)^{2/3}}. \quad (46)$$

The time interval for the photon to return is approximately $(2d_0 + \Delta d)/c$ and thus the distance traversed is

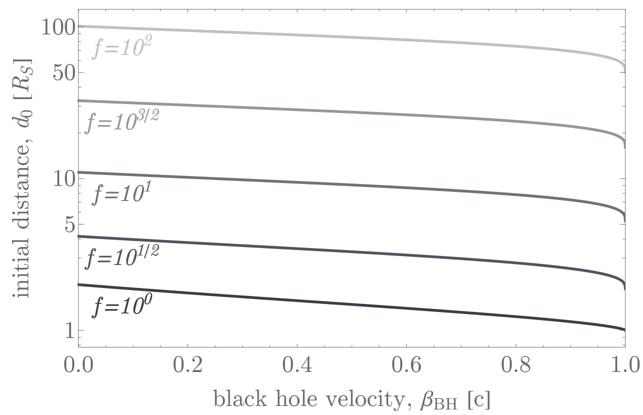

Figure 4. Initial stand-off distance from the black hole such that gravitational redshift effects are f times smaller than those of Doppler effects when using the halo drive. The approximation $d_0 \gg R_S$ found analytically in the limit of $\beta_{\rm BH} \to 0$ generally holds up well except for extreme cases.

$$\Delta d \simeq \frac{2d_0\beta_2}{1-\beta_2}. \quad (47)$$

The gravitational redshift from d_0 to $d_0 + \Delta d$, when the photon returns, is given by

$$\nu_{f,\rm corr} \simeq \nu_f \sqrt{\frac{(d_0+\Delta d)(d_0 - R_S)}{d_0(d_0+\Delta d - R_S)}}. \quad (48)$$

This effect can be considered to be insignificant if the relativistic Doppler correction made in the previous subsection far exceeds the change caused by the gravitational red shift, i.e. when

$$1 - \sqrt{\frac{(d_0 + \frac{2d_0\beta_2}{1-\beta_2})(d - R_S)}{d(d + \frac{2d_0\beta_2}{1-\beta_2} - R_S)}} \ll 1 - \sqrt{\frac{1-\beta_2}{1+\beta_2}}. \quad (49)$$

Solving for d_0 in the limit of $\beta_1 \to 0$, one can show that equates to the condition that $d \gg R_S$, where R_S is the Schwarzschild radius. If the ratio between the RHS and the LHS of the above is labelled as f, then Figure 4 demonstrates that this argument works well even for relatively high $\beta_{\rm BH}$. Accordingly, it is argued that the terminal velocity derived in Equation (45) is accurate so long as $d_0 \gg R_S$, in which case additional effects such as changes in the relative binary position leading to time-dependent gravitational redshifts can also be safely ignored.

3.6. *Numerical tests*

Throughout this work, it has been assumed that the principle of ensemble equivalence described in Kipping (2017) also holds here, although this has not been tested. The problem closely resembles that described in Kipping (2017) and thus generally it is expected to hold. Further, Section 3.3 showed that a single-photon acceleration produced the same results as that of the double-photon acceleration curve. Of course, a single photon emitted with an energy comparable to the rest mass of the spacecraft is not feasible (or indeed desirable) and generally implementation would involve the emission of a large sequence of lower energy photons to produce a more gradual acceleration.

It is therefore worthwhile to test whether the terminal velocity predicted from a single photon model indeed equals that when a large number of sequential emissions are performed instead. Using the equations described throughout this work, a calculation was performed for the acceleration for N photons of equal frequencies set to $\nu = (mc^2\kappa_{i1})/(hN)$, where κ_{i1} is set to the value derived earlier necessary to achieve terminal velocity in the case of a single photon. If the principle holds, then the final velocity after numerically integrating N sequential steps should equal the terminal velocity (to within floating point precision).

As shown in Figure 5, it is easy to verify that the principle holds and more over it is possible to accurately predict the terminal velocity of the spacecraft using our formulae.

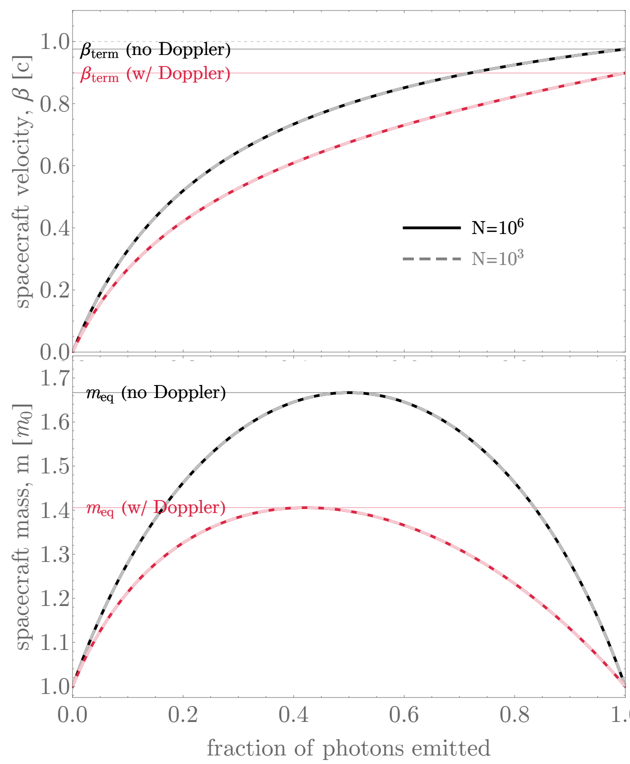

Figure 5. Numerical tests of the principle of ensemble equivalence for $\beta_{\rm BH} = 0.8$. As expected, photons can be released either in a small number at high energies or in a large number of equivalent cumulative energy, but the results are the same. The dashed lines show the predictions from our earlier derivations in the case of a single photon assumption. Accounting for Doppler shifts leads to significant changes in the results.

4. DISCUSSION

4.1. *Response of the binary & observational signatures*

The halo drive causes a spacecraft of essentially arbitrary mass (so long as $m \ll M_\odot$) to accelerate up to relativistic speeds (for suitably compact binaries) without losing any fuel in the process[1]. At face value, this makes remote detection of halo drives seemingly impossible. But, there is no such thing a free lunch and of course something here has lost energy and that's the binary itself. By the time it reaches terminal velocity, the spacecraft has increased its energy from mc^2 to $mc^2\gamma_{\rm term}$, and accordingly one can write that the binary must have lost an energy of

$$\begin{aligned}\Delta E &= (\gamma_{\rm term} - 1)mc^2,\\ &= \frac{mc^2}{2}\left(\frac{\left((1+\beta_{\rm BH})^{2/3} - (1-\beta_{\rm BH})^{2/3}\right)^2}{(1-\beta_{\rm BH}^2)^{2/3}}\right).\end{aligned} \quad (50)$$

The binding energy of a binary system is given by

$$E = -\frac{GMM_2}{2a} + \mathcal{O}[\tfrac{1}{c^2}], \quad (51)$$

where a is the binary separation and M_2 is the mass of the secondary component. If the binary evolves from a to $a - \Delta a$ as a result of the kick, then one may show that to first-order in $\Delta E/E$

$$\frac{\Delta a}{a} = -\frac{2a\Delta E}{GMM_2}. \quad (52)$$

[1] Note that solutions do exist for moving $m \sim M_\odot$ via an alternative mechanism, as described in Shkadov (1987); Forgan (2013).

If one writes that $a = \tilde{a}GM/c^2$ (i.e. in half Schwarzschild radii), then

$$\frac{\Delta a}{a} = -\tilde{a}\frac{m}{M_2}\left(\frac{\left((1+\beta_{\mathrm{BH}})^{2/3}-(1-\beta_{\mathrm{BH}})^{2/3}\right)^2}{(1-\beta_{\mathrm{BH}}^2)^{2/3}}\right). \tag{53}$$

Expanding the ΔE term to second-order in β_{BH}, accurate to 3% for all $\beta_{\mathrm{BH}} < 0.5$, yields

$$\frac{\Delta a}{a} \simeq -f\frac{m}{M_2}\beta_{\mathrm{BH}}^2. \tag{54}$$

This reveals that the binary will be kicked into a slightly eccentric orbit with the periapsis position located at the extraction point, with the new semi-major axis shrinking by that described by Equation (54). In general, these changes are small for all $m \ll M_2$ or effectively all $m \ll M_\odot$.

A civilization using a network of binaries may not only accelerate from them but also decelerate upon return, thus potentially undoing the slight distortions made to the binary. Even so, the binary temporarily spends time at closer semi-major axis where gravitational radiation is more effective and thus one still expects elevated merger rates to result.

One-way trips, perhaps from a central hub, would lead to an even higher rate of binary in-spiral on-top of the natural gravitational radiation. If journeys are made isotropically, an eccentric binary may not result but accelerated in-spiral would persist. However, only a discrete set of highways exist between galactic binary black holes and thus the distortions can never be perfectly isotropic meaning that excess eccentricity would likely persist.

4.2. *From infinitesimal to finite beams*

One effect ignored in the earlier derivation is that it was assumed that the beam has an infinitesimal width. In reality, the beam has a finite width and that width will diverge in a physically real system. It is therefore critical that the beam divergence over the entire path length is less than the size of the spacecraft's receiver, L, else significant energy losses would occur.

Beams will diverge due to two effects. The first of these is via diffraction, and for a diffraction limited beam one expects the width to diverge after a distance $2d$ to

$$W_r = W_t + \frac{2\sqrt{2}d\lambda}{D_t}, \tag{55}$$

where D_t is the diameter of the transmitter and W denotes the width of the beam at reception and transmission. If the spacecraft has a physical width of L, then one requires

$$\lambda \ll \frac{LD_t c^2}{2\sqrt{2}(d/R_S)2GM}, \tag{56}$$

or

$$\lambda \ll 120\,\mu\mathrm{m}\left(\frac{L}{D_t}\right)\left(\frac{D_T}{10\,\mathrm{m}}\right)^2\left(\frac{d}{100R_S}\right)^{-1}\left(\frac{M_\odot}{M}\right). \tag{57}$$

In the neighborhood of the black hole, within a hundred Schwarzschild radii, it should be easy to produce collimated electromagnetic radiation at such wavelengths. This is can be extended to much greater distances if the receiver is much larger than the transmitter ($L \gg D_T$). This latter point is particularly relevant because the halo drive is able to accelerate effectively arbitrarily large masses up to β_{term} (so long as $m \ll M_\odot$) allowing for extremely large (e.g. planet-sized) vehicles. Ultimately, diffraction can be overcome by simply using shorter wavelength light, or even particle beams. For this reason, although diffraction is an unavoidable effect, it could be mitigated against unless halo drives are attempted at extreme distances where it may become impractical to emit/absorb such high energy radiation.

A second effect that leads to beam divergence comes from essentially a tidal effect. Consider a beam which has finite width and is emitted at a single angle, δ, tailored such that the center of the beam will perform a boomerang geodesic (e.g. using the method described in Section 2.1). Photons emitted slightly off to the side beam's center will encounter the black hole at slightly different impact parameters. Since the beam angle is chosen such that only the center line performs a boomerang, then the edges will saddle the separatrix and experience distinct deflection angles, leading to the effect of achromatic beam divergence.

Let's say that the edge of the beam is offset from the center by a distance $W_t/2$. A light ray emitted from this point crosses the radial line between the black hole and the center of the beam at a distance $d + W_t/(2\tan\delta)$. Accordingly, the correct angle this photon should be emitted at to perform a boomerang is not δ, but rather (using the result from Figure 2):

$$\delta_{\mathrm{edge}} = \frac{\delta_0}{d + \frac{W_t}{2}\cot\frac{\delta_0}{d}}. \tag{58}$$

Accordingly, the beam would potentially miss the spacecraft upon return. The key problem is that the photons at the edge of the beam were emitted at the wrong angle, δ, whereas the correct boomerang angle would have been δ_{edge}.

This point reveals that the problem actually stems from the way in which the beam was chosen to be setup - a planar source such that the entire beam has the same initial emission angle. For this reason, the divergence is not unavoidable in the same sense as diffraction is, but rather is primarily an engineering problem that could be surmountable through careful beam shaping (see Dickey 2003). The purpose of this work is not to provide an actual blueprint for the halo drive, but rather merely highlight that no physical barrier exists to prevent such a scheme. Nevertheless, one possible solution could be a large number of micro-emitters with independent actuators that would be combined to form the overall beam, where each micro-emitter has a unique angular displacement to correct for the effect, analogous to how adaptive optics corrects wavefront errors in the Earth's atmosphere using individual actuators. Clearly, such a system would require a very advanced control system to make the necessary calculations for each actuator, but again there's no obvious physical barrier to overcoming this problem.

4.3. *Ignored effects*

It is important to highlight several approximations made in this work. The purpose of this paper is to introduce the concept of using halos as described, and thus several small effects were ignored to facilitate the calculations that are briefly discussed here.

First, this work has assumed that extremely efficient absorption of the photon is assumed by the spacecraft upon reception. An idealized system needs to be able to recycle the photons with thermal losses (see Slovick et al. 2013) much smaller than the total energy transferred to the spacecraft, ΔE.

A second effect ignored is the energy to overcome the gravitational potential energy of the binary in order to escape the system. Tacitly, it was assumed that the velocities achieved far exceed the escape velocity from the initial standoff distance. Requiring ΔE of Equation (50) to be much greater than the gravitational potential energy of a binary where $M_2 = qM$, one may show that

$$\frac{d}{R_S} \gg \left(\frac{1+q}{2}\right)\left(\frac{(1-\beta_{\mathrm{BH}}^2)^{2/3}}{\left((1+\beta_{\mathrm{BH}})^{2/3}-(1-\beta_{\mathrm{BH}})^{2/3}\right)^2}\right),$$
$$\frac{d}{R_S} \gg \left(\frac{27\beta_{\mathrm{BH}}^{-2}-22}{48}\right), \tag{59}$$

where on the second line, right hand bracket has been Taylor expanded to first-order as well as assuming $q \sim 1$. For low β_{BH}, such as $\beta_{\mathrm{BH}} = 0.05$, this requires a large stand-off distance of a couple of thousand Schwarzschild radii. In the mildly relativistic scenario of $\beta_{\mathrm{BH}} = 0.2$, standoff distances greater than around a hundred Schwarzschild radii would make the gravitational potential energy factor much smaller than the gained energy. Nevertheless, it could be worthwhile to include this generally small contribution in future work.

A third assumption is that the circumbinary environment is devoid of opaque material that would lead to beam losses. For example, an accretion disk around the black hole would certainly make it a sub-optimal target for a halo drive. Accordingly, if one requires compact binaries for relativistic acceleration, the other component would need to be another black hole or neutron star to avoid mass transfers forming a disk.

4.4. *Other applications of the halos*

Numerous earlier works have highlighted the potential use of black holes for advanced technological applications (e.g. see Crane & Westmoreland 2009; Inoue & Yokoo 2011) and the halo drive provides another example.

Although not the focus of this work, it is worth highlighting that halo drives could have other purposes besides from just accelerating spacecraft. For example, the back reaction on the black hole taps energy from it, essentially mining the gravitational binding energy of the binary. Similarly, forward reactions could be used to not only decelerate incoming spacecraft but effectively store energy in the binary like a fly-wheel, turning the binary into a cosmic battery.

Another possibility is that the halos could be used to deliberately manipulate black holes into specific configurations, analogous to optical tweezers. This could be particularly effective if halo bridges are established between nearby pairs of binaries, causing one binary to excite the other. Such cases could lead to rapid transformation of binary orbits, including the deliberate liberation of a binary.

It is also highlighted that acceleration could be performed in a two-body process where the source is a very massive emitter in the system but the halo strikes a second nearby and lower mass vehicle. This vehicle could then be accelerated to even faster velocities than the terminal velocity computed earlier. Such a system would lead to the more massive source also experiencing a kick back into a higher orbit, as well transferring some fraction of its initial mass to the accelerated vehicle. Thus, the system would have a finite lifetime before the accelerator would reach very large orbital radii where halos would become difficult to establish via diffraction constraint of Equation (57).

4.5. *Kerr metrics*

This work has focused on halo drives being applied to a Schwarzschild black hole (Schwarzschild 1916) in a compact binary system. However, it is hypothesized here that lone, isolated Kerr black holes (Kerr 1963) could likely serve the same function. By riding along the frame dragged spacetime surrounding the black hole, light should be blue shifted (in the case of same sense revolution), permitting the rotational energy of the black to be tapped[2]. This joins the numerous ways previously proposed to extract energy from Kerr black holes, such as the Penrose process (Penrose & Floyd 1971), superradiance with amplifying incident waves for various fields (Zeĺdovich 1972; Bardeen et al. 1972; Starobinsky 1973; Starobinsky & Churilov 1973; Teukolsky & Press 1925) and the Blandford-Znajek process (Blandford & Znajek 1997). Calculation of the Kerr-case was beyond the scope of this work but would be an interesting problem for the future.

5. CONCLUSIONS

The search for intelligence amongst the cosmos is often guided by considering the possible activities of hypothetical advanced civilizations and the associated technosignatures that would result (e.g. Dyson 1960; Lin et al. 2014; Korpela et al. 2015). At the same time, there is growing interest in developing the means for humanity to take our first steps into becoming an interstellar civilization (e.g. *Breakthrough Starshot*; see Parkin 2018). These two enterprises can often overlap, since advanced propulsion systems may lead to observable technosignatures (e.g. Guillochon & Loeb 2015). Along these lines, this work has considered how an advanced civilization might utilize the light sailing concept to conduct relativistic and extremely efficient propulsion.

[2] It is highlighted that Cramer (1997) calculate boomerang geodesics for Kerr black holes but the blue shift effect was not considered.

The proposed system is that a spacecraft emits a collimated beam of energy towards at a black hole at a carefully selected angle, such that the beam returns to the spacecraft - a so-called boomerang geodesic (Stuckey 1993). If the black hole is moving towards the spacecraft, as could be easily accomplished by exploiting a compact binary, this halo of particles will return with a higher energy (and momentum). This energy is then transferred to the spacecraft allowing for acceleration. Overall then, the halo drive transfers kinetic energy from the moving black hole to the spacecraft by way of a gravitational assist.

The analysis presented assumes the halo is photonic, but the beam could be comprised of massive particles too and achieve the same effect. Either way, the system described echoes the Dyson (1963) slingshot, except that the spacecraft does not physically slingshot around the compact object, but rather let's the light beam do the slingshot on its behalf.

An appealing aspect of the halo drive is that no fuel is spent. The spacecraft gradually gains energy during its initial acceleration and then discharges that energy for further acceleration up to terminal velocity - the speed at which the spacecraft returns to its original mass.

The terminal velocity of the spacecraft is 133% the black hole's speed, to first-order. Critically, this velocity in not sensitive to the mass of the spacecraft, with the only assumption being that said mass is much less than that of the black hole. Accordingly, a major advantage of the halo drive is that Jupiter-mass spacecraft could be accelerated to relativistic speeds.

Beam divergence due to tidal effects on a finite beam width could be mitigated by careful beam shaping. Divergence due to diffraction is not expected to lead to noticeable losses for large spacecraft using optical lasers within a hundred Schwarzschild radii. Nevertheless, for this reason, the system is argued to be impractical at distances much greater than this, thereby necessitating relatively expedient acceleration.

An advanced civilization utilizing such a system would first have to have achieved interstellar flight to journey towards the nearest suitable BH. They could then could use BHs in binary systems as way-points throughout the galaxy, of which there are likely $\mathcal{O}[10^7]$ in the Milky Way (Reggiani & Meyer 2013), serving as both acceleration and deceleration stations. Alternatively, they could use the larger population of BHs which do not reside in compact binaries (Elbert et al. 2017) via their proper motions, although this would not permit for such high velocities.

Each departure from a binary in a particular direction kicks the binary into a slightly eccentric orbit and accelerates it's in-spiral merger rate. In principle, each arrival from the same direction would undo this effect leading to no observable signature. However, finite time differences between the departure and arrival would cause the binary to spend time at a tighter semi-major axis than it would naturally, during which time it would experience more rapid gravitational radiation in-spiral. Accordingly, a possible technosignature of the halo drive would be an enhanced rate of black hole binary in-spiral, versus say their neutron star counterparts.

6. ACKNOWLEDGEMENTS

DMK is supported by the Alfred P. Sloan Foundation. Thanks to Nick Stone, Zephyr Penoyre, Zoltan Haiman, Jerry Ostriker, Janna Levin, Avi Loeb and Claes Cramer for helpful conversations in preparing this manuscript. I would also like to thank Bill Stuckey for his correspondence regarding gravitational mirrors, and Michael Hippke and Duncan Forgan for constructive comments on an early draft of this paper. I am also grateful to the anonymous reviewers for their helpful comments.

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